Identifying, estimating and quantifying uncertainty in hydrological modeling

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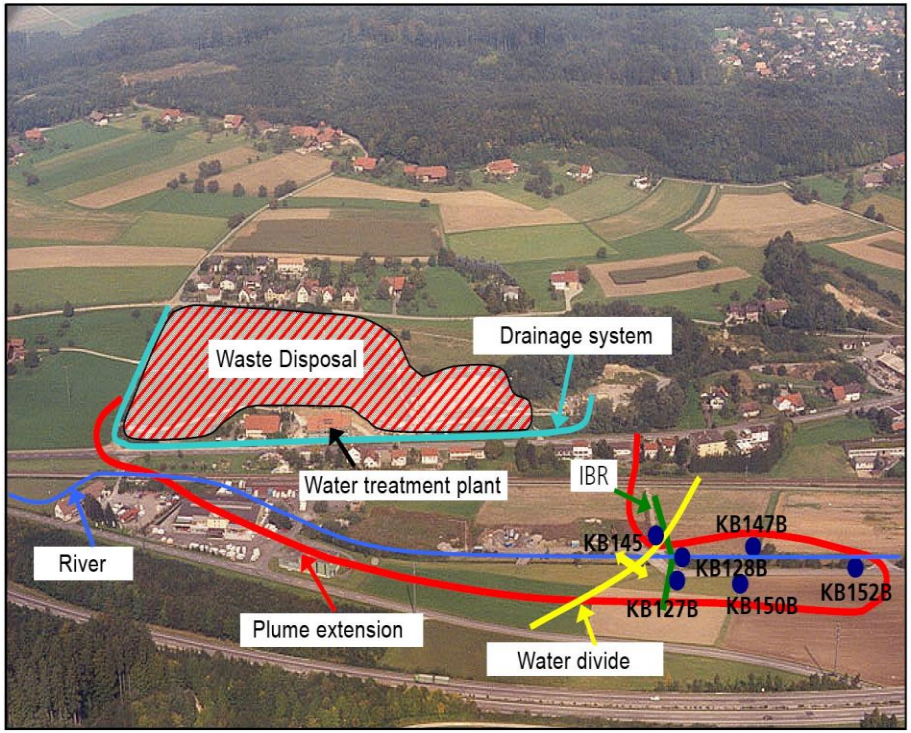
Kölliken waste landfill site

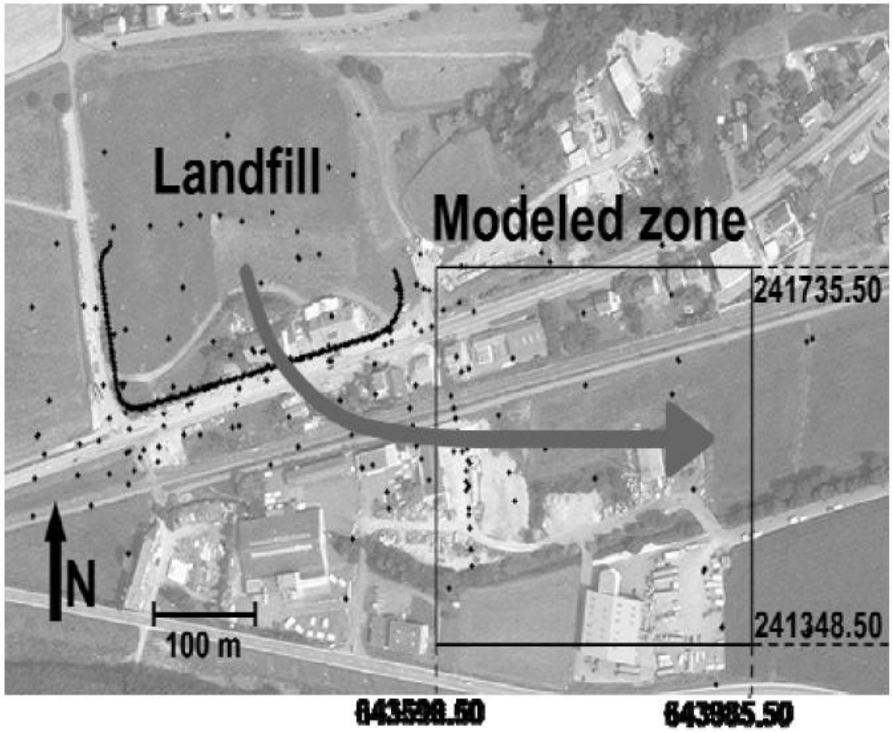


- Absence of impervious layer to prevent leakage, no good drainage
- Remediation measures – drilling drainage tunnel and wells
- Heterogeneous character of the site

Figure from Mariethoz et al. (2008)

- Waste disposal site
- Drainage system
- Plume extension

Kölliken waste landfill site



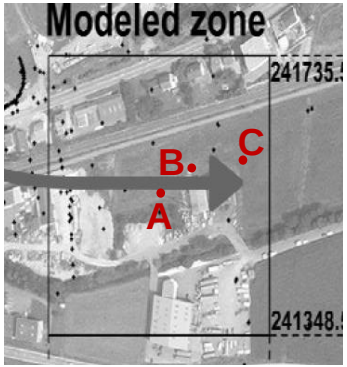
- Absence of impervious layer to prevent leakage, no good drainage
- Remediation measures – drilling drainage tunnel and wells
- Heterogeneous character of the site
- Small part of the plume could not be recovered by pumping
- Modelled zone includes plume extension that was not captured

Figure from Mariethoz et al. (2008)

- Waste disposal site
- Drainage system
- Plume extension
- 245 boreholes, 219 measurements along the drainage tunnel, each 5-50m deep

Decision uncertainty, risk measure and costs

- Decision comes with a cost (avoid risk aversion)
- How many new wells are needed? And where?
- Assume that a fixed pumping rate is proposed, would you support approval?
- This has possible impacts on overlying alluvial aquifer, and the local communities and farmland



Decision uncertainty, risk measure and costs

What matters?

- Cost effectiveness, impact on stream base flows
- **Different outcomes of concern**

Which hydrological predictions matter?

- Link the outcomes to hydrological impacts
- **Different predictions of interest**

Does prediction uncertainty matter?

- Consider the sensitivity of errors in prediction of those hydrological impacts
- **Different sensitivities to uncertainty**

➢ How do we deal with problems when there are multiple stakeholders with different interests in a common problem?

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Hydrology road map

Sparse **data**

Incomplete/defective **models**

BUT, **decisions** have to be made

- **Uncertainty definition:**
  - Characterization of data
- **Uncertainty propogation:**
  - Prediction using models
- **Uncertainty quantification:**
  - Identify and quantify input-response relations
  - Sensitivity analysis

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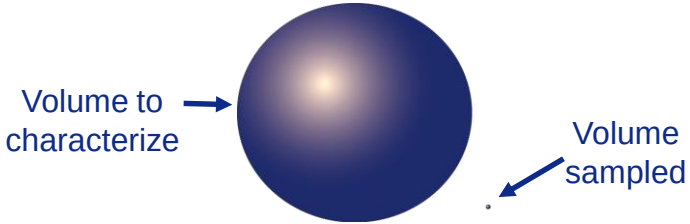
Understanding uncertainty: Some uncertainty is reducible

Reducible uncertainty/ambiguity

- it is a potential deficiency that is due to the lack of knowledge
  - can arise from assumptions introduced in the derivation of the mathematical model
  - other examples are parameter and deicison uncertainty,
  - not naturally defined in a probabilistic framework
  - can lead to a strong bias of predictions



➢ Introduce uncertainty in a way that defines statistical dependencies in a clear way

Understanding uncertainty: Other uncertainty is irreducible

Irreducible uncertainty/hydrological variability

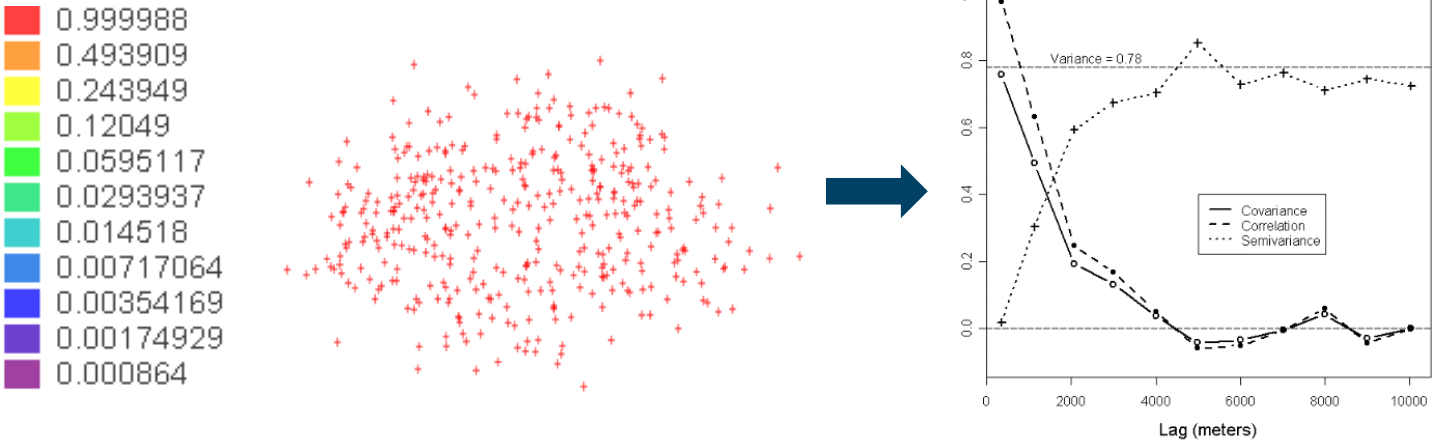
- it is the inherent variability present in the system/environment
  - not strictly due to a lack of knowledge
  - naturally defined in a probabilistic framework
  - temporal and **spatial** (material properties, precipitation, river flow, water quality etc.)



➢ Seek ways to better characterize the variability in a probabilistic framework

Variography: From empirical data to theory

- Characterization of spatial correlation
- Theoretical quantities that you cannot observe
- Need to be estimated from available data (empirical covariance/semivariograms)



Variography is a way of quantifying the structure, where you fit a spatial-dependence model to your data

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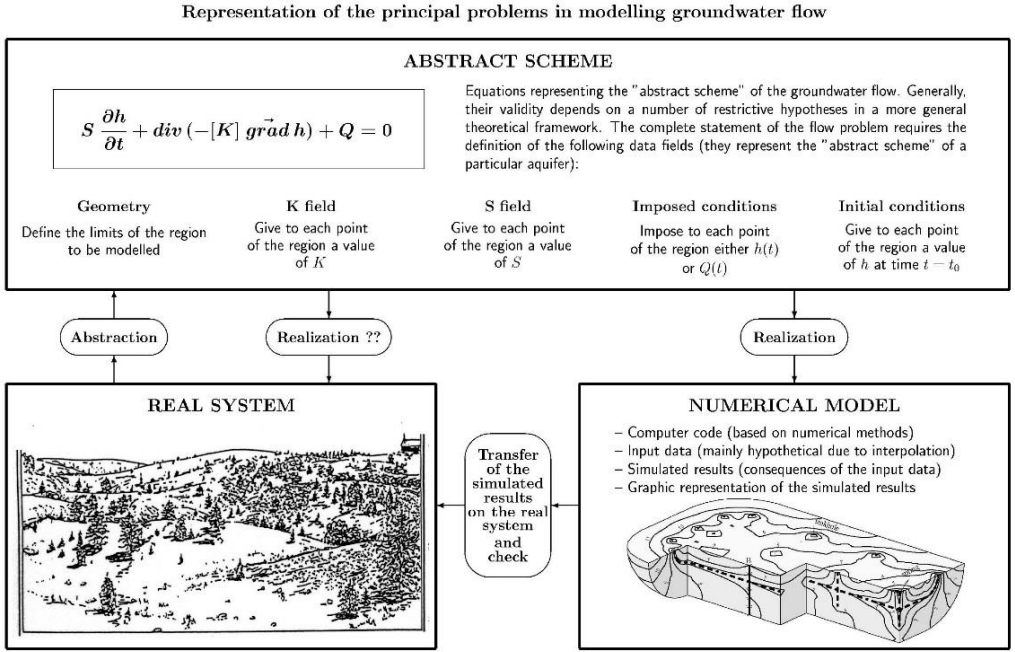
Numerical Models - reconstruction of reality



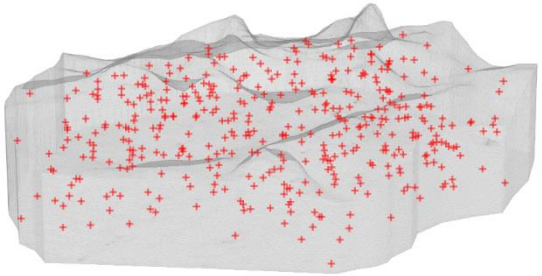
- From little knowledge to a schematic representation
- Limited data of physical parameters
  - indirect estimation,
  - interpolation/extrapolation
- Realization of a computer code based on numerical methods allowing eqns to be defined in the abstract scheme
- Transfer of the simulated results onto the real system

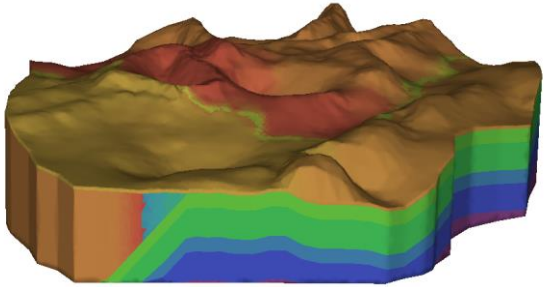
Are the simulated results valid?

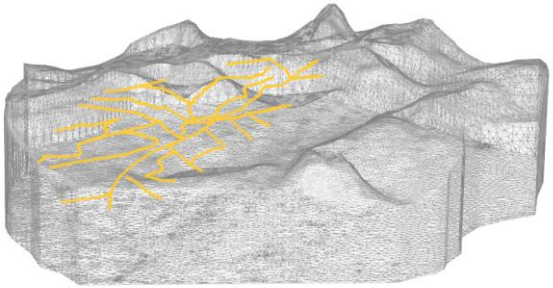
- Yes, but...

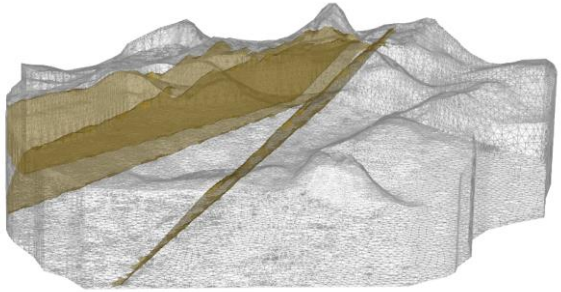
The meaningful transfer of results to reality requires uncertainties on the data to appear as uncertainties on the results

Models – simplified representations of complex things





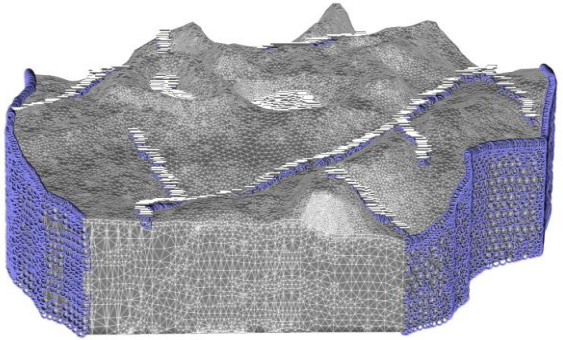


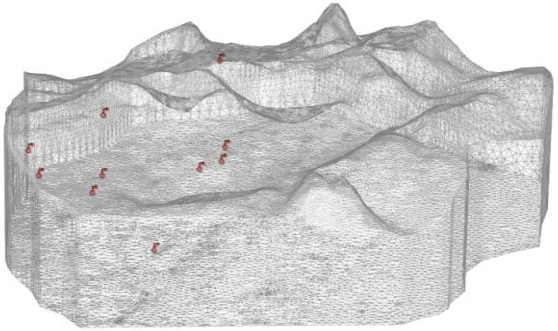


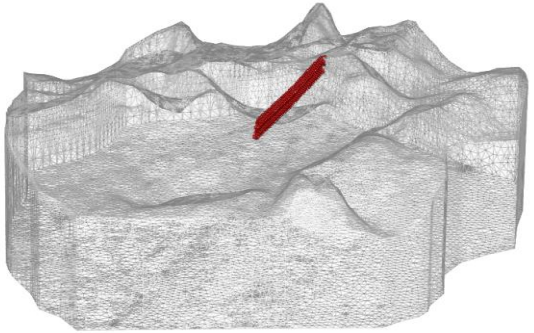
Modeling for decision making:

- Models with many sources of uncertainty
- Geological structures
- Package – FEFLOW
- Processes – included in the analysis

Identifying boundaries (internal and external)

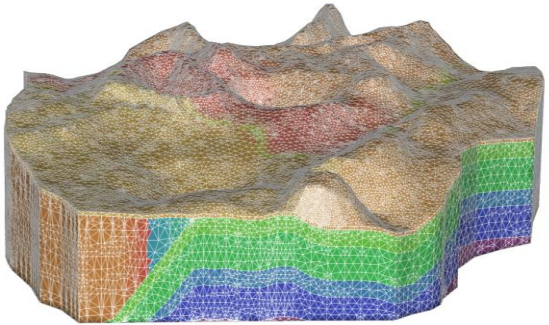






Modeling for decision making:

- models with many sources of uncertainty
- Geological structures
- Package – FEFLOW
- Processes – included in the analysis
- Boundary conditions and forcing terms
- **Parameters (spatial/temporal correlation)**

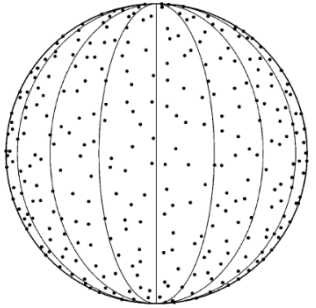


Turning Band Method – for Gaussian simulations

- Perform multi-d simulations at the CPU cost of 1-d simulations
- Define a covariance model $C_X(\boldsymbol{x})$
- Determine the uni-dimensional covariance C_Y for each line on the unit sphere S_d
- Random process on line generations
- Simulate the multidimensional random field by summing contributions from random process on uni dimensional lines

LINE GENERATION PROCESS

- Generate a fixed number of on-line realizations $Z_{1,i}(\boldsymbol{x} \cdot \boldsymbol{u})$ based on normal distributions and unidimensional covariance. $\boldsymbol{u}$ is uni dimensional vector on line i, Z_1 are the values of realizations on line i



- Dependence on number of lines

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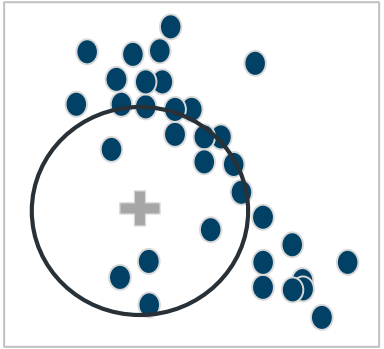
Conditioning via Kriging

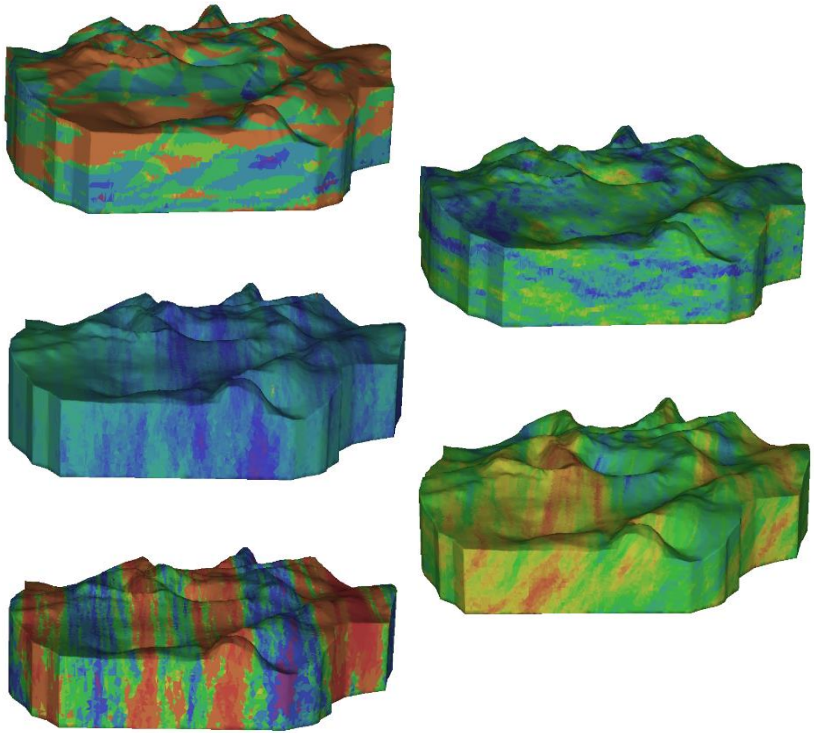
- Draw an unconditional simulation at the target location and data locations via the turning bands method.
- Compute the residuals (deviations) between the data values and simulated values at the data locations.
- Perform a simple kriging of the residual from its values at data locations.
- Add the result to the unconditioned realization

NEIGHBOURHOOD SEARCH

- More spatial correlation(relative to distance)
- Neighbourhood shape
- Number of neighbours
- Weights assigned to each neighbour within the block

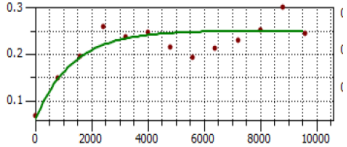


Stochastic realizations for input parameter fields



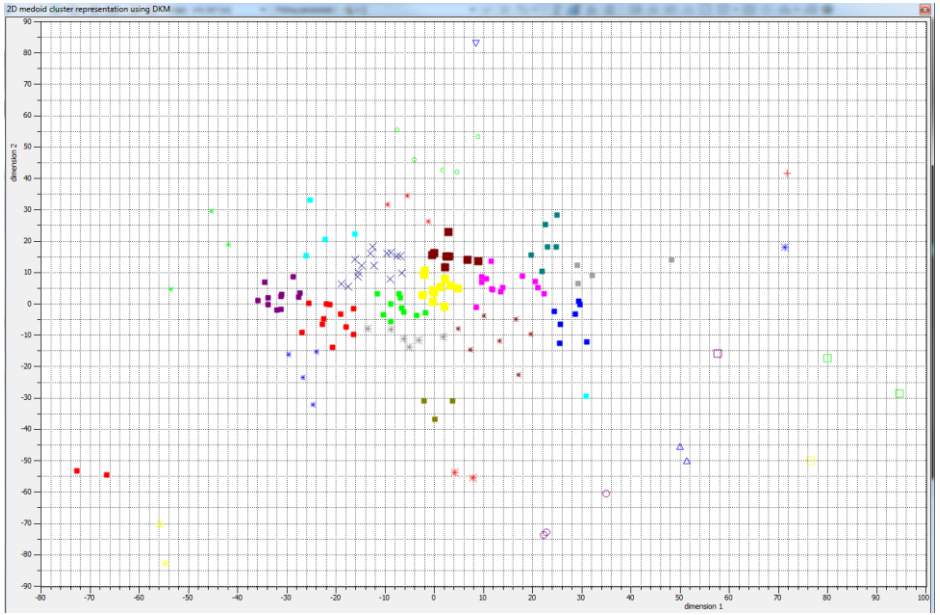
- Grid: ~1 mio elements
- 400 point measurements
- Data transformation using NST

- Turning band method
- Residual based kriging
- Exponential covariance model



- Back transformation using nst
- 30 realizations each (5 sets = 150 samples)

100 inputs; Now what?



- Data uncertainty quantified with ‚distance measures‘
- Each realization viewed as a „dot“

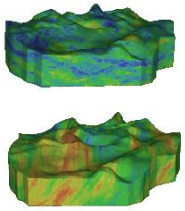
- Detect similarities/redundancies
  - Discriminatory data
- Dimensionality reduction (MC)
- Medoid realizations can be used to span the space of uncertainty

➢ Decisions are often clearer in a ‚decision space‘ – Distance Kernel Method (Caers et al.)

Uncertainty propogation

Perform simulations accounting for the uncertainty represented as randomness

- Define an abstract probability space (Ω, $\boldsymbol{P}$)
- Introduce uncertain input as random quantities $\boldsymbol{k}(\omega)$
- The original problem becomes stochastic with solution $\boldsymbol{u}(\omega)$

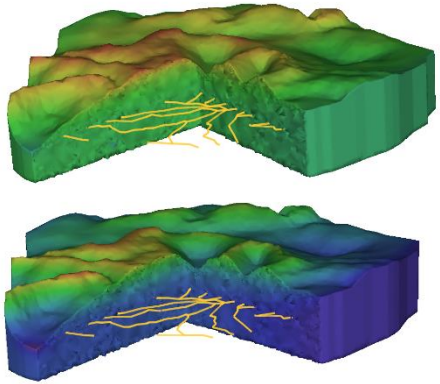


Stochastic input

Computational model + Transfer functions

FEFLOW

Stochastic output



Uncertainty can affect the bc's, geometry, forcing terms or operators in the computational model and numerical solvers

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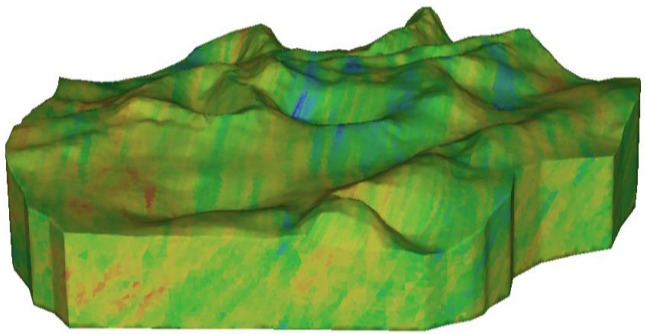
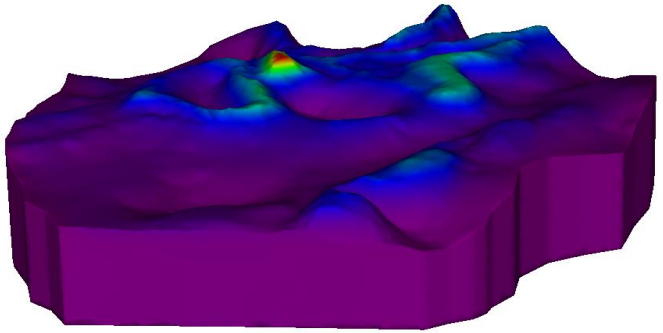
Sensitivity analysis

- Define a validation metric to compare uncertain quantities
  - Variability: how the variability in an output is linked to the input
- Simulation results do NOT compare directly to data





- Allows to build a ranking of the input sources which might dominate the response of the system
- Investigate correlations between input and output

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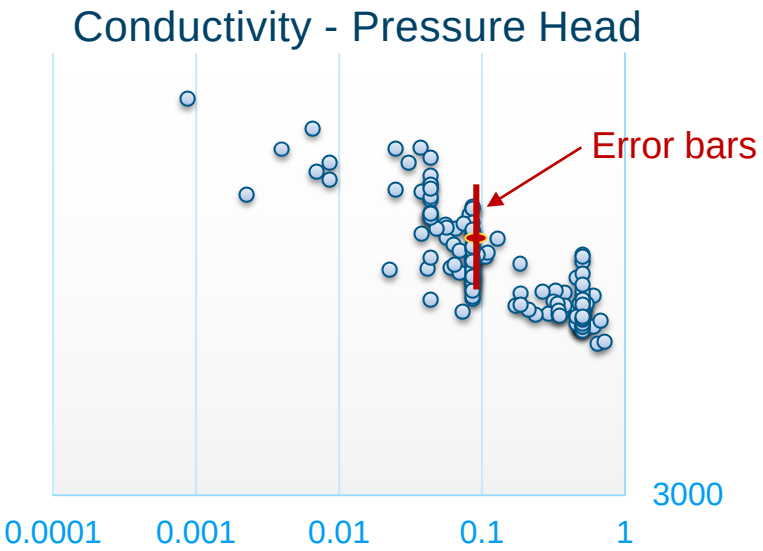
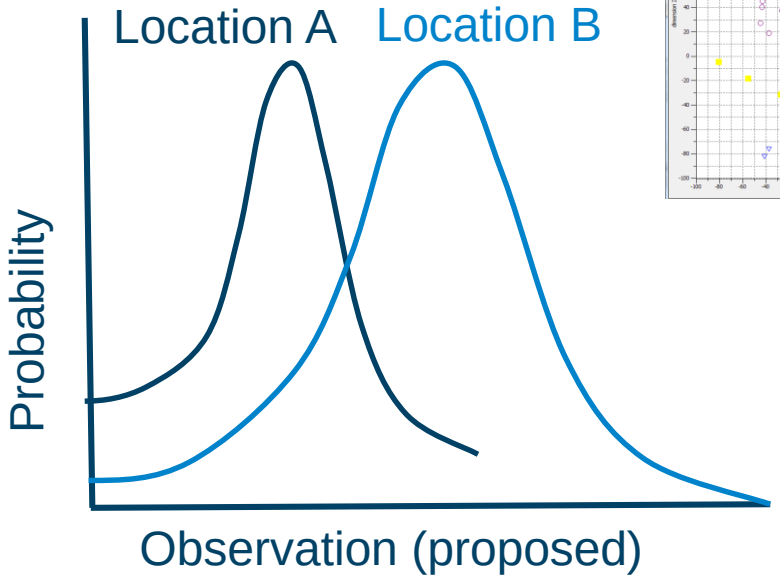
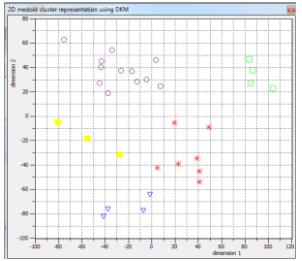
Input-response relations / probability distributions

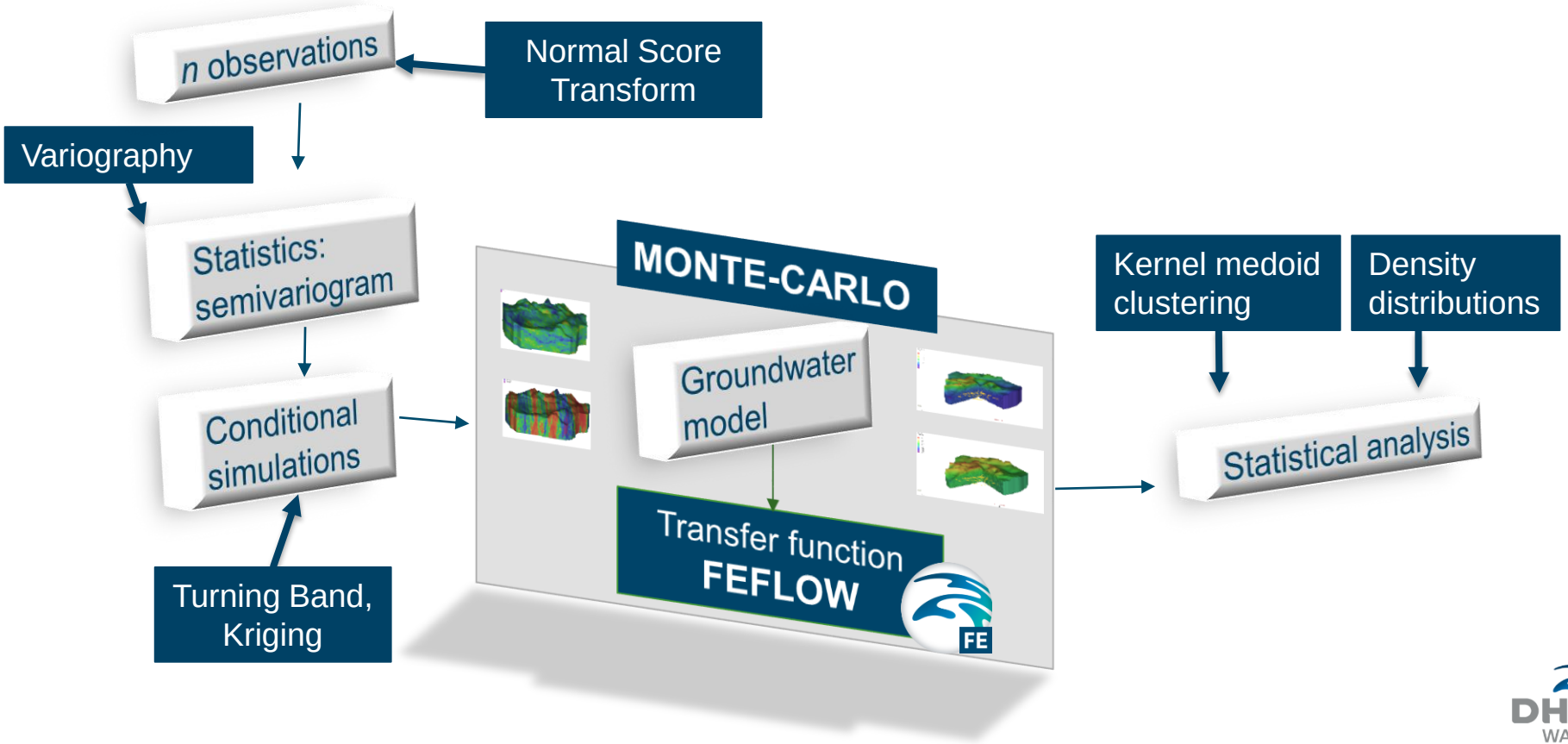






- Every realization is a possible location of the well to be added to the network
- Generate a pdf of expected variations
- Link them to pdf of the predictions of interest and chose the well with the highest variance

Uncertainty workflow interfaced with FEFLOW



Redefining Objectives

Robust decision making requires the use of an ensemble of models, conditioned on discriminatory data

Objectives of a Modeling and Simulation tool

- Assist the user in decision making
- Enable the user to **characterize uncertainty**
  - For **credible** decision making
  - Contribute usefully towards water resource management issue

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